The Set of the Real

Mathematical Implications of the Metaphysics of René Guénon

by Peter Samsel

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> *…more than any other science, mathematics thus furnishes a particularly apt symbolism for the expression of metaphysical truths to the extent that the latter are expressible…*[1]
>
> René Guénon

René Guénon, the seminal founder of the Traditionalist School, was also perhaps its preeminent metaphysician. More particularly, he was the plenary expositor of a metaphysics through which mathematical conceptualization runs like a golden thread. As Frithjof Schuon, another remarkable metaphysician, has observed, "Guénon was like the personification, not of spirituality as such, but uniquely of metaphysical certainty; or of metaphysical self-evidence in mathematical mode…"[2] As is well known, Guénon's primary intellectual formation prior to his plunge into the esoteric was that of mathematics, which he both studied and taught for many years.[3] In this, he followed a well established tradition of linkage between mathematics and metaphysics extending back to such figures as Pythagoras and Plato.[4] Indeed, mathematics in Guénon's hands is employed as an adequate symbolism for metaphysical realities, most notably in *The Symbolism of the Cross*, while elsewhere he takes pains to metaphysically rectify mathematics of certain modernist distortions, most notably in *The Metaphysical Principles of the Infinitesimal Calculus*. In both these endeavors, he sought to reinvigorate mathematics as the handmaid of metaphysics.

What has not been previously recognized is the underlying mathematical structure of what is perhaps his central metaphysical work, *The Multiple States of the Being*. The evident reason for this lack of recognition is that he nowhere indicates that there is such a formal structure. Nevertheless, this structure is integral to his entire presentation, as we shall endeavor to demonstrate. In particular, the very language of his metaphysical description finds its exact parallel in the mathematical theory of sets, more specifically the earliest version, developed by the Russian-born mathematician Georg Cantor, known as naïve set theory.[5] This recognition leads to two intriguing questions: First, did Guénon know of Cantor's set theory and deliberately integrate its perspectives into his metaphysical writings? Second, if he did in fact do so, what are the implications for the veracity of his metaphysical exposition?

Set theory, rather than evincing the long history of development characteristic of most subdisciplines of mathematics, begins precipitously with the work of Cantor,[6] whose writings on the subject span the 1870s to late 1890s, his final double treatise on the topic being published in 1895 and 1897. Guénon, a precocious student of mathematics in his youth, terminated his formal academic studies in mathematics in 1906 at roughly the age of twenty.[7] Nevertheless, it is clear that he remained interested in mathematics for much of his life, although the degree to which he kept abreast of developments in the discipline is uncertain. Judging by the bare ordering of dates above, it is certainly possible that Guénon may have been aware of Cantor's work, although this is by no means conclusive.

Another line of inquiry is to examine possible references to Cantor or set theory in Guénon's writings. Although there is no reference to set theory to be found in Guénon's core metaphysical texts, there is a single reference to Cantor,[8] addressing the question of infinity, to which Cantor also made major contributions. Certainly, much of Guénon's invective toward modern notions of multiple infinities can be seen as directed largely at Cantor's ideas. This criticism would, it seems, necessarily assume some familiarity on Guénon's part with Cantor's work on infinities. Curiously, Cantor's study of infinities dovetails quite closely with his development of set theory,[9] suggesting a possible awareness by

Guénon of Cantor's work on sets. Nevertheless, this would somewhat ironically imply Guénon's categorical rejection of one aspect of Cantor's work, combined with his mute but complete acceptance of another aspect. Further, where he does invoke or discuss specific mathematical subdisciplines—such as analytic geometry or the calculus—he is typically not hesitant to identify them by name.

The question is—without further biographical data—impossible to settle conclusively, but it is by no means implausible that Guénon was to some degree aware of set theory, either through Cantor or those who followed him. If he was so aware, it is difficult to escape the conclusion that he incorporated set-theoretic conceptions into his metaphysics, either consciously and deliberately, or possibly in a more semi-conscious and intuitive manner. Having said this, it is by no means impossible that Guénon could have conceived his metaphysical exposition entirely independently of such knowledge, as the conceptions with which naïve set theory deals are extremely fundamental and intuitive. If we assume that Guénon in fact had some knowledge of naïve set theory how should this affect our consideration of his metaphysics? It would seem that there are two possible interpretations: either to consider his metaphysical exposition as in some way undermined by its close formal association with mathematical theory, upon which the metaphysics is in some sense partly derivative, or alternatively, to view the mathematics as a suitable and adequate vehicle for the expression of metaphysical truths.

The former interpretation suffers from having the historical weight of metaphysical speculation against it, as mathematics has often been employed as a suitable metaphor for metaphysical realities. This same observation in turn supports the latter interpretation, which may be further supported by detailed examination of Guénon's writings. To cite just two examples, Guénon associates the act of integration in the calculus as an adequate symbol of the realization of the Self: "…integration and other operations of the same kind will thereby veritably appear as symbols of metaphysical 'realization' itself."[10] The adequation of the "summing up" of integration with the realization of the totality of the Self in all is readily apparent. Similarly, the notion of a condition upon a set and its elements—fundamental in Cantor's analysis—is an adequate symbol of the notion of limiting conditions found in Advaita Vedanta: "…particular and limiting conditions (*upadhis*), which are looked upon as so many bonds."[11]

Quite apart from the question of origination and association that Guénon's metaphysical exposition might bear with respect to set theory, there is another compelling reason for the investigation of set-theoretic concepts and formalism as they bear upon his metaphysics: the clarification of his teaching. If metaphysics may serve as a "saving truth", this salvific character is somewhat blunted in his writings due to a certain opacity that often accompanies the limpidity of thought for which he is justly recognized. As a long-time friend of his observed: "Guénon's books demonstrate that particular confusion which does not exclude clear-sightedness or clear expression, but yet results in one's being quite unable to extract any precise ideas from them."[12] This is, perhaps, too often the unfortunate reaction to Guénon's metaphysical writings. Against such an outcome, the formal clarity elucidated by a study of the underlying mathematical structure of his ideas may provide a precious key to the deeper understanding and assimilation of his metaphysical teachings. Of course, this key cannot simply be given to the reader, but must be earned: there is no "royal road" to set theory, but to the degree possible, we endeavor in what follows to render the way easy and level for the non-mathematically inclined.

A Brief Primer on Naïve Set Theory

Set theory, or the mathematical theory of sets, is one of the axiomatic foundations of mathematics. As such, its foundational character yields the fortunate virtue of it being intuitive and straightforward in its fundamental application; set theory is, in its most basic sense, simply a way of reasoning with precision regarding collections of things. A set itself is simply a collection of elements. Consider a string quartet as a possible set; its elements would be the parts of the quartet: first violin, second violin, viola, cello. In mathematical parlance, this may be described as,

$$\textit{String Quartet} = \{\textit{first violin}, \textit{second violin}, \textit{viola}, \textit{cello}\}$$

Here, *String Quartet* is a set, and the items listed in { } are the elements of the set. In mathematical practice, neither the ordering nor the duplication of elements is taken into account. One might also consider more abstract sets, such as,

$$A = \{2, 4, 6, 7\} \text{ and } B = \{1, 3, 5, 7\}$$

If an element belongs to a set, we may express its membership with the notation $\in$; conversely, if an element does not belong to a set, we use the notation $\notin$. Thus,

$$2 \in A \text{, but } 3 \notin A$$

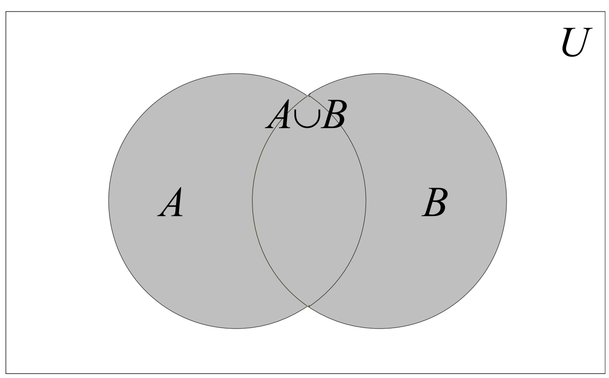
and similarly,

$$1 \in B \text{, but } 2 \notin B$$

Set Operations

There are three basic operations between sets: *union*, *intersection* and *difference*. The *union* of two sets is the set comprised of all elements found in *either* or *both* sets. Thus the union of *A* and *B* may be expressed as,

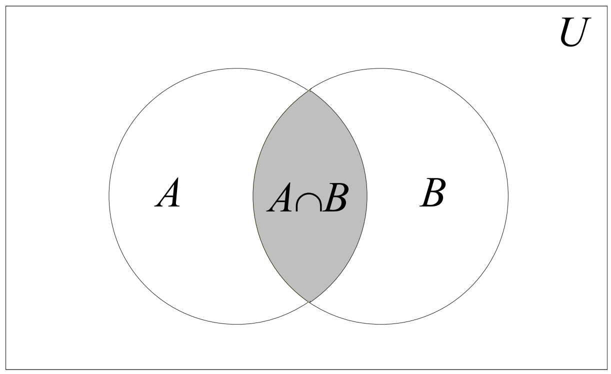
$$A \bigcup B = \{1, 2, 3, 4, 5, 6, 7\}$$



The shaded section is the union of A and B, $A \bigcup B$

In contrast, the *intersection* of two sets is the set comprised of all elements common to *both* sets. Thus the intersection of *A* and *B* may be expressed as,

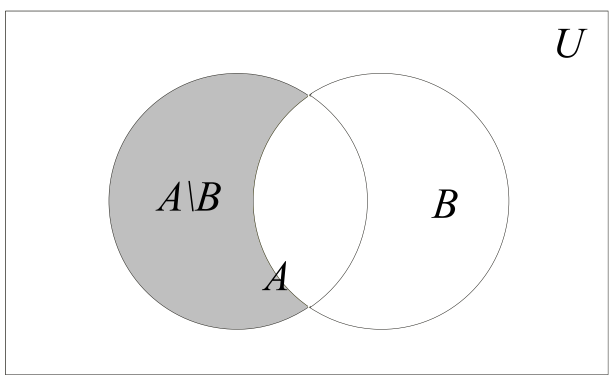
$$A \bigcap B = \{7\}$$



The shaded section is the intersection of A and B, $A \bigcap B$

Finally, the *difference* between two sets is the set comprised of all the elements in the first set that are not in the second set. Thus the difference of *A* and *B* may be expressed as,

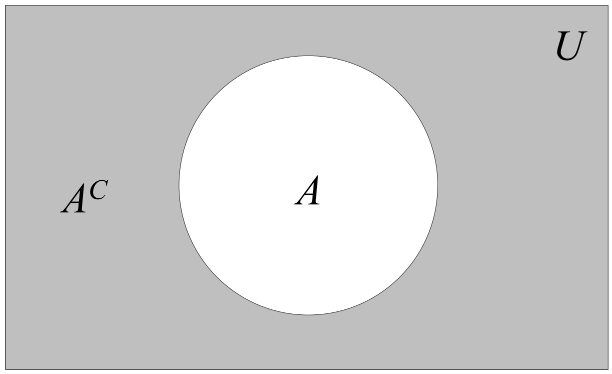
$$A \setminus B = \{2,4,6\}$$



The shaded section is the difference of A and B, $A \setminus B$

A set that is empty of any elements is termed the *empty set,* or *null set*, and may be written as $\{\}$ or ϕ. Conversely a set containing all possible elements is termed the *universal set* and is typically written as *U*. The *complement*, A^C, of a set *A*, is the set of all elements not found in A. It may be defined in terms of the difference of set A and the universal set U as,

$$A^C = U \setminus A$$



The shaded section is the complement of A, A^C

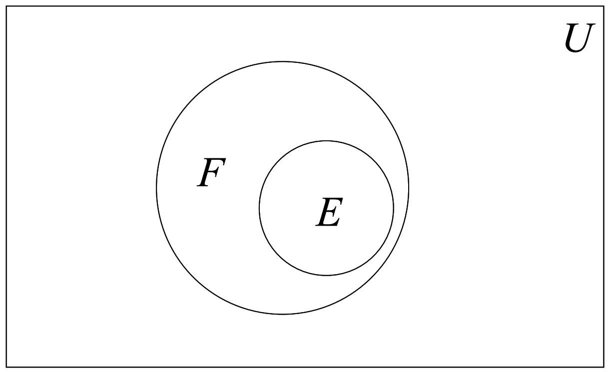
Set Relationships

There are four possible relationships between sets: *subset/superset*, *identical*, *conjoint* or *disjoint*. Two sets E and F are respectively *subset* and *superset* if all elements of E are in F, but not all elements of F are in E. We may formalize this in terms of the union and intersection of the two sets, where we use the notation $\subset$ to indicate subset, as,

$$E \subset F \textit{ if } E \bigcup F = F \neq E \textit{ and } E \bigcap F = E \neq F$$

As an example,

$$\text{for } E = \{g, h, d\} \text{ and } F = \{h, d, g, f\},\ E \subset F$$



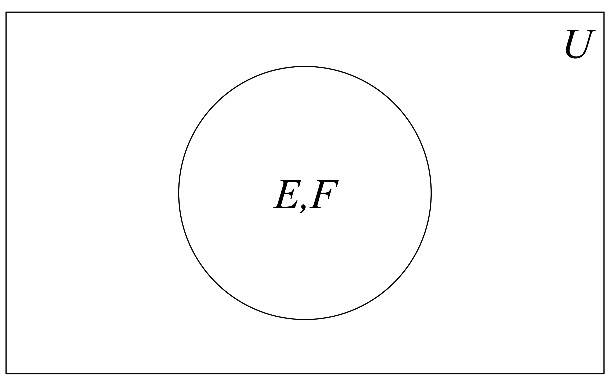
E is a subset of F, F is a superset of E, $E \subset F$

Two sets E and F are *identical* if all elements of E are in F, and all elements of F are in E; that is, if all elements are common between the two sets. We may formalize this in terms of the union and intersection of the two sets, where we use the notation = to indicate identity, as,

$$E = F \text{ if } E \bigcup F = E \bigcap F = E = F$$

As an example,

for $E = \{f, g, h, d\}$ and $F = \{h, d, g, f\}$, $E = F$



E is identical to F, $E = F$

If E or F are undetermined, one may consider the combined case where E may be either a subset of or identical to F. We may formalize this in terms of the union and intersection of the two sets, where we use the notation $\subseteq$ to indicate "subset or identity", as,

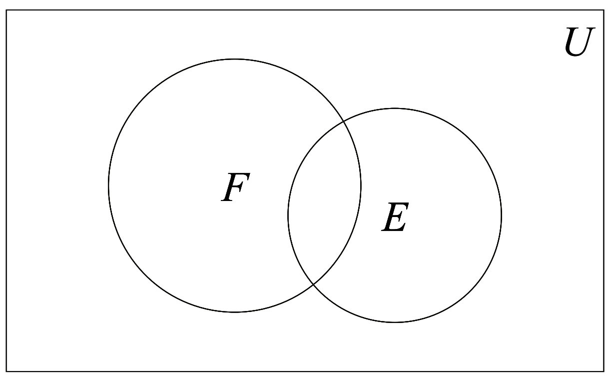
$$E \subseteq F \text{ } if \text{ } E \bigcup F = F \text{ } and \text{ } E \bigcap F = E$$

Two sets E and F are *conjoint* if the intersection of E and F is neither empty, nor identical to either E or F; that is, if the two sets share some elements without either sharing all. We may formalize this in terms of the intersection of the two sets as,

$$E,F \text{ } conjoint \text{ } if \text{ } E \bigcap F \neq \phi \text{ } and \text{ } E \bigcap F \neq E \text{ } or \text{ } F$$

As an example,

for $E = \{f, g, h, d\}$ and $F = \{h, d, m, n\}$, E,F *conjoint*



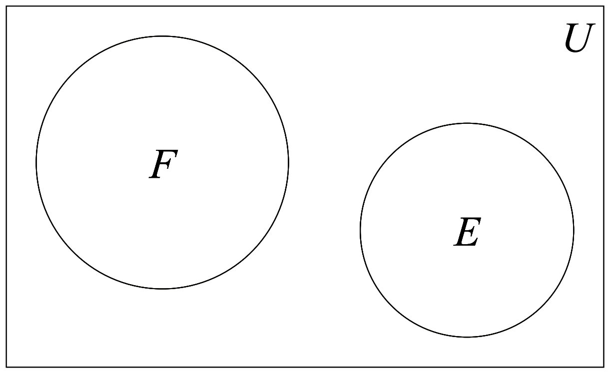
E and F partially intersect, E,F *conjoint*

Two sets E and F are *disjoint* if the intersection of E and F is empty. We may formalize this in terms of the intersection of the two sets as,

$$E,F \text{ } disjoint \text{ } if \text{ } E \bigcap F = \phi$$

As an example,

for $E = \{f, g, e, j\}$ and $F = \{h, d, m, n\}$, E,F *disjoint*



E and F do not intersect, E, F *disjoint*

Set Builder Notation

If we consider the set of integers, *I*, this may be expressed as,

$$I = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

where the ellipses indicate the infinite continuation of the elements of the set, both negatively and positively. If we wished to specify the set of only positive integers, I_P, we could write,

$$I_P = \{1, 2, 3, ...\}$$

Another way of considering this would be to recognize that the set of positive integers is just the set of integers subject to the condition that its members are restricted to be greater than zero. We may express this in terms of *set builder notation* as,

$$I_P = \{x \in I : x > 0\}$$

Here, x is an arbitrary variable that is to be evaluated over all possible elements, the first statement '$x \in I$' indicates that x must be an element of the set of integers, the notation : means "such that", and '$x > 0$' is the condition that x must satisfy. We might restate this in ordinary language as:

The set of positive integers is the set containing all elements of the set of integers such that they satisfy the condition that they are greater than zero.

One may expand this notation to include multiple conditional terms. We may introduce a *conjunctive* connective $\wedge$, analogous to the word "and", which joins two conditions, both of which must be satisfied. Additionally, we may introduce a *disjunctive* connective $\vee$, analogous to the word "or", which joins two conditions, either or both of which may be satisfied. We may employ these logical connectives to express multiple conditions that must be satisfied by the members of a given set. Thus, the set of positive integers less than ten, $I_{P<10}$, may be expressed as,

$$I_{P<10} = \{ x \in I : (x > 0) \wedge (x < 10) \}$$

Again, we might restate this in ordinary language as:

The set of positive integers less than ten is the set containing all elements of the set of integers such that they satisfy the conditions that they are greater than zero and less than ten.

These logical connectives in set builder notation are fundamentally related to the fundamental set operations, as may be seen in the following expressions:

$$A \bigcup B = \{ x \in U : (x \in A) \vee (x \in B) \}$$

The union of sets A and B is the set containing all possible elements such that they satisfy the condition that they are either members of set A or set B or both.

$$A \bigcap B = \{ x \in U : (x \in A) \wedge (x \in B) \}$$

The intersection of sets A and B is the set containing all possible elements such that they satisfy the condition that they are both members of set A and set B.

$$A \setminus B = \{ x \in U : (x \in A) \wedge (x \notin B) \}$$

The difference of sets A and B is the set containing all possible elements such that they satisfy the condition that they are members of set A but not of set B.

$$A^C = \left\{ x \in U : x \notin A \right\}$$

The complement of set A is the set containing all possible elements such that they satisfy the condition that they are not members of set A.

A point made clear by set-builder notation is that, while a set is *comprised* of elements, is may be essentially *defined* by the condition or conditions that it satisfies. To generalize, we may term the condition satisfied by A as C_A; similarly, the condition satisfied by B as C_B. Then,

$$A = \left\{ x \in U : C_A \right\},\ B = \left\{ x \in U : C_B \right\}$$

For instance, if C_A is '*x integer* $\wedge$ $x > 0$' and C_B is '*x integer* $\wedge$ $x < 10$', then A and B are,

$$A = \left\{ 1, 2, 3, \ldots \right\},\ B = \left\{ \ldots, 7, 8, 9 \right\}$$

Note that here we have placed the requirement that x be integer to the right of the colon as a condition, rather than to the left of the colon as an initial set restriction; either construction is valid. We may then reexpress the union and intersection of A and B as,

$$A \bigcup B = \left\{ x \in U : C_A \vee C_B \right\}$$

The union of sets A and B is the set containing all possible elements such that they satisfy either conditions C_A or C_B or both.

$$A \bigcap B = \left\{ x \in U : C_A \wedge C_B \right\}$$

The intersection of sets A and B is the set containing all possible elements such that they satisfy both conditions C_A and C_B.

If we return to the examples of C_A and C_B above, then the disjunction $C_A \vee C_B$ is '*x integer* $\wedge$ ($x > 0 \vee x < 10$)', but ($x > 0$

$\vee$ *x < 10*) is unrestricted, since any number will be either greater than 0 or less than 10. Thus $C_A \vee C_B$ is simply '*x integer*' and

$$A \bigcup B = \{x \in U : x \in I\} = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

We next consider the conjunction $C_A \wedge C_B$, which is '*x integer* $\wedge$ *x > 0* $\wedge$ *x < 10*'; this is restricted to integers between zero and ten, and

$$A \bigcap B = \{x \in U : (x \in I) \wedge (x > 0) \wedge (x < 10)\} = \{1, 2, 3, \ldots 7, 8, 9\}$$

In this particular instance, it is clear that the two sets *A* and *B* are conjoint, as they intersect, yet neither is contained by or identical to the other. However, an alteration of the conditions defining *A* and *B* could readily alter their respective relationship. Thus, if C_A were '*x integer* $\wedge$ *x < 0*' and C_B were '*x integer* $\wedge$ *x > 10*', then *A* and *B* would be disjoint, while if C_A were '*x > 3* $\wedge$ *x < 9*' and C_B were '*x > 0* $\wedge$ *x < 10*', then *A* would be the subset of *B*. In short, the nature of C_A and C_B will dictate the relationship between the sets *A* and *B*.

An additional observation regarding set operations and set relationships is that the union of sets is, in general, expansive, while the intersection of sets is, in general, restrictive. More precisely,

$$A \bigcup B \supseteq A, B$$

The union of A and B is equal to or a superset of A and B.

$$A \bigcap B \subseteq A, B$$

The intersection of A and B is equal to or a subset of A and B.

The generality of set theory lies in its very simplicity, for it essentially rests on the very broad notions of *collectio*n and *membership*. Although we have made use of numbers in the preceding examples, there is nothing particularly numeric or quantitative regarding set theory. Indeed, the elements of a set may be numbers, things, persons or metaphysical categories, just as their associated conditions may be quantitative, qualitative or abstract.

Set Theory and the Metaphysics of René Guénon

In discussing Guénon's metaphysics, one must first begin with the notion of the metaphysical Infinite, Guénon's term for the transcendent metaphysical Principle. The metaphysical Infinite is expressed by Guénon most concisely as "that which has absolutely no limits whatsoever."[13] As such, it "cannot admit of any restriction, which presupposes that it be absolutely unconditioned and undetermined."[14] Closely related to the metaphysical Infinite is the notion of universal Possibility, which is,

> an aspect of the Infinite, from which it is in no way and in no measure distinct;…it is nothing other than the Infinite itself envisaged under a certain aspect—insofar as it is permissible to say that there are aspects to the Infinite.[15]

Universal Possibility and the Universal Set

Viewed in terms of the language of set theory, and, in particular, set-builder notation, every set is defined in terms of the condition or conditions that must inherently limit it. To define a set is to limit it, yet the metaphysical Infinite, as without limits, cannot be so defined, indeed "can only be expressed in negative terms by reason of its absolute indetermination."[16] In set-theoretic terms, this immediately suggests the universal set U, which bears a striking correlation to the metaphysical Infinite taken in aspect as universal Possibility. Indeed, the definition of U that may be constructed in set-builder notation,

$$U = \{x : x = x\}$$

imposes no limit of any kind, the condition that *every possible element equal itself* being at once trivial and tautological. One might give another definition in terms of the language of set operations: the universal set U is the set that, for any arbitrary set S apart from U, $S \subset U$. That is, U is a superset of every possible set apart from itself. Alternatively, one may define U in terms of set membership: $x \in U$ for all possible elements x.

In this abstract sense, the set-theoretic notion of a universal set is very close to the notion of universal Possibility. However, in practice, U is invariably specified to be bounded in some way. This may be done in part for the sake of convenience, as when mathematicians constrain U to be the set of real numbers. More fundamentally, however, this is done in order to avoid certain inherent paradoxes that otherwise arise. The most famous, *Russell's Paradox*, states that if the universal set U contains every possible set, then it must contain the set of all sets that are not members of themselves; but this set is neither a member of itself nor not a member of itself, and therefore cannot exist. The paradox implies that the naïve notion of the universal set as containing all possible sets cannot be supported without contradiction. Such an issue clearly weakens the association of the set-theoretic notion of the universal set with the metaphysical notion of universal Possibility, but there is a much more significant question that is raised as well: does the invalidation of the naïve conception of the universal set also invalidate Guénon's notion of the metaphysical Infinite and universal Possibility and therefore potentially his entire metaphysical exposition?

There are two responses that may be given to this possible conclusion. First, such paradoxes as Russell's Paradox rely upon Aristotle's law of the excluded middle, or noncontradiction, as axiomatic. Yet, where questions of metaphysics are concerned, the law of the excluded middle is known to perform badly or not at all. Consider the Vedantic *mahavakyas*: *Brahman satyam, jagan mithya*; *Sarvam idam brahma*: Brahman is real, the world is not; all this is Brahman. Taken together, they clearly violate the law of the excluded middle, and yet to not take them together is to fall into metaphysical error. Second, it is not the case, in Guénon's exposition, that everything contained within universal Possibility must therefore also necessarily exist or even have the potentiality for existence. Thus "the set of all sets that are not members of themselves" may be contained in universal Possibility without thereby existing, thus avoiding its paradoxical implications. More generally, the set proposed by Russell is but one of a vast collection of self-refuting objects, such as the Buddhist "son of a barren woman", to which might be added self-refuting statements, such as the truth of the Heraclitean proposition that "knowledge is impossible", all of which may find their place, inexistent, within universal Possibility.

The Sets of Non-Manifestation and Manifestation

We may thereby associate universal Possibility with the naïve notion of the universal set U as unlimited and all-containing. The elements comprising the set U may then be understood as the unlimited indefinitude of possibilities contained within universal Possibility. Within universal Possibility, the first distinction that may be made is the "distinction between the possibilities of manifestation and the possibilities of non-manifestation, both being included equally and by the same right in total Possibility."[17] The possibilities of manifestation are, of course, those that are manifested, whereas the possibilities of non-manifestation are those that are unmanifested. As Guénon expresses:

> …every possibility that is a possibility of manifestation must necessarily be manifested by that very fact…inversely, any possibility that is not to be manifested is a possibility of non-manifestation.[18]

Manifestation and non-manifestation are, necessarily, the domains encompassing these respective possibilities. Passing to the language of set theory, it is clear that, just as the various possibilities are elements within the universal set of universal Possibility, so the domains of manifestation and non-manifestation are subsets containing the respective possibilities associated with each. Further, from their conceptual description, it is clear that, since a possibility is *either* manifest or non-manifest, that their union must be equivalent to universal possibility, while they must be nonintersecting. But this is simply the description of a set and its complement, so that we may express the manifest domain as the set M and the non-manifest domain as its complement M^C. Their relationship may then be given as,

$$M \bigcup M^C = U \,; \;\; M \bigcap M^C = \phi$$

We may also define these domains in terms of set-builder notation, where the respective conditions to be evaluated are '*p is manifest'*; '*p is unmanifest'*, as,

$$M = \{p \in U : p \text{ is manifest}\}; \;\; M^C = \{p \in U : p \text{ is unmanifest}\}$$

The set of manifestation is the set containing all possibilities such that they satisfy the condition of being manifest; *the set of non-manifestation is the set containing all possibilities such that they satisfy the condition of being unmanifest*.

Set Theory and the Domain of Manifestation

While we may apply the language of set theory to this first distinction between the manifest and unmanifest, this language is not suitable for describing the structure of the domain of nonmanifestation, as here the basic distinctiveness and determination necessary to apply the fundamental set-theoretic notions of *element*, *collection* and *conditionality* are lacking. As Guénon states, regarding the unmanifest:

> There can be no question of a multiplicity of states, since this domain is essentially that of the undifferentiated and even of the unconditioned; the unconditioned cannot be subject to the determinations of the one and of the multiple, and the undifferentiated cannot exist in a distinctive mode.[19]

It is to the domain of manifestation that we may apply this language, as here one may find distinction, multiplicity and conditional relation. It is here that are found an indefinite multiplicity of possibilities and modes of manifestation. Again, as Guénon clarifies:

> [Existence] comprises the indefinite multiplicity of the modes of manifestation, for it contains them all equally by the very fact that they are all equally possible, this possibility implying that each one of them must be realized according to the conditions proper to it.[20]

Here, Guénon denotes existence as a collection—or set—of modes or possibilities of manifestation, each of which may be realized, or not, in the presence of certain conditions. The suitability of set theory, and in particular set-builder notation, for the specification of this domain should be readily apparent. If a set of possibilities within manifestation is defined, as in set-builder notation, by the conditions that the elements in the set satisfy, then each of these elements must be mutually compatible with the multiple conditions in question. Here, Guénon—following Leibniz—makes use of the term "compossibles" to denote possibilities that are mutually compatible with multiple conditions:

> Compossibles are in fact nothing but possibilities that are mutually compatible, that is to say whose union in a complex whole introduces no contradictions into the latter.[21]

In fact, there is some ambiguity in Guénon's use of the term: whether it applies to the possibilities themselves or to the conditions that determine them. To a certain degree, this ambiguity is inherent; consider squareness: this designation may refer to an attribute of a given possibility, just as it may refer to the condition that such a possibility must satisfy. If we combine squareness with, say, materiality, these may be seen to form compossibles, as there are possibilities—such as window panes—that may mutually satisfy both conditions. In contrast, squareness and roundness, in combination, cannot be compossibles, since no existent possibility may mutually satisfy both attributes or conditions:

> So, taking first an example of a particular and extremely simple order, a "round square" is an impossibility because the union of the two possibles "round" and "square" in the same figure implies contradiction; but these two possibles are nonetheless also realizable, for the existence of a square figure obviously does not preclude the simultaneous existence of a round one in the same space, any more than it does any other conceivable geometrical figure.[22]

We may clarify these relationships in the language of set-builder notation. We first define a number of terms:

M	the set of manifestation
S_{square}	the set of manifested possibilities possessing the attribute of squareness
S_{round}	the set of manifested possibilities

	possessing the attribute of roundness
$S_{material}$	the set of manifested possibilities possessing the attribute of materiality
C_{square}	the condition of squareness
C_{round}	the condition of roundness
$C_{material}$	the condition of materiality

Then, by definition, we may express:

$$S_{square} = \{p \in M : C_{square}\};$$
$$S_{round} = \{p \in M : C_{round}\};$$
$$S_{material} = \{p \in M : C_{material}\}$$

where each set is defined in set-builder notation in terms of its associated condition. If we next consider the above conditions in combination, we may express the set of compossibles satisfying both squareness and materiality as the intersection of the two sets S_{square} and $S_{material}$, which in turn relate to the conjunction of the conditions C_{square} and $C_{material}$:

$$S_{square} \cap S_{material} = \{p \in M : C_{square} \wedge C_{material}\}$$

This set of compossibles satisfying both squareness and materiality is, of necessity, a subset of both S_{square} and $S_{material}$, as follows from the general relation regarding intersections:

$$S_{square} \cap S_{material} \subset S_{square} \text{ , } S_{material}$$

This set is clearly not empty, as the conditions C_{square} and $C_{material}$ are not mutually exclusive. In contrast, the set of compossibles satisfying both squareness and roundness,

$$S_{square} \cap S_{round} = \{p \in M : C_{square} \wedge C_{round}\}$$

is in fact empty,

$$S_{square} \cap S_{round} = \phi$$

as the conditions C_{square} and C_{round} *are* mutually exclusive. Expressed in terms of set relationships, we may say that S_{square} and $S_{material}$ are conjoint, as are S_{round} and $S_{material}$, while S_{square} and S_{round} are disjoint.

Conditions, Possible Sets and Compossible Sets

Of course, manifestation comprises an indefinitude of conditions, some of which may be stipulated—such as form, extension, temporality, materiality and so forth—while the majority are quite likely inconceivable, given the very narrow slice of manifestation that we experience. Nevertheless, we may abstract the notion of a condition and define the set of conditions pertaining to manifestation as:

$$C_a, C_b, C_c, C_d, C_e, C_f, C_g, \ldots$$

where we employ arbitrary subscripts and deliberately leave the character of each condition unspecified. A set of possibilities satisfying each condition may abstracted similarly:

$$S_a, \mathrm{S}_b, \mathrm{S}_c, \mathrm{S}_d, \mathrm{S}_e, \mathrm{S}_f, \mathrm{S}_g, \ldots$$

where

$$S_a = \{p \in M : C_a\};\ S_b = \{p \in M : C_b\};\ \ldots$$

Any two given sets of manifested possibilities, such as S_a and S_b, will be related in terms of one of the four fundamental set relationships—subset/superset, identical, conjoint or disjoint—where the exact relationship will depend on the characters of the associated conditions C_a and C_b. Broadly speaking, two conditions are compatible if their associated sets are related as subset/superset, identical or conjoint; the conditions are incompatible if their associated sets are disjoint.

We may also abstract the notion of a set of compossibles, such as that satisfying both conditions C_a and C_b, as:

$$S_a \cap S_b = \{p \in M : C_a \wedge C_b\}$$

Of course, such a set need not be limited to the conjunction of only two conditions, but may be defined in terms of the conjunction of an arbitrary number of conditions, equivalent to the intersection of their associated sets. Thus, the set of compossibles satisfying conditions C_a, C_c, C_f and C_g may be expressed as:

$$S_a \cap S_c \cap S_f \cap S_g = \{p \in M : C_a \wedge C_c \wedge C_f \wedge C_g\}$$

We may equivalently express this in terms of summation notation, as:

$$\bigcap_{i \in \{a,c,f,g\}} S_i = \{p \in M : \wedge_{i \in \{a,c,f,g\}} C_i\}$$

where i is an index variable evaluated over the members of the set of conditional subscripts $\{a, c, f, g\}$ and the intersection and conjunction operators are now treated as summations:

$$\bigcap_{i \in \{a,c,f,g\}} S_i = S_a \cap S_c \cap S_f \cap S_g ;$$
$$\wedge_{i \in \{a,c,f,g\}} C_i = C_a \wedge C_c \wedge C_f \wedge C_g$$

If we represent a given set of conditional subscripts—corresponding to the conditional delimitation of a compossible set—with an arbitrary Greek letter for distinction, we may then express this set of compossibles more compactly as:

$$\bigcap_{i \in \gamma} S_i = \{p \in M : \wedge_{i \in \gamma} C_i\}$$

where γ is taken in this instance as denoting $\{a, c, f, g\}$. We might further notate such a set of compossibles by the notation:

$$S_\gamma = \bigcap_{i \in \gamma} S_i$$

where S_γ is the compossible set delimited by the set of conditions whose subscripts are denoted by γ.

To briefly review, the indeterminate possibilities found within manifestation may be subject to an indeterminate number of conditions: C_a, C_b, C_c, C_d, C_e, C_f, C_g,… The set of possibilities that satisfy a given condition, such as C_a, is expressed in terms of the same subscript, as S_a. There are an indeterminate number of such sets of possibilities—S_a, S_b, S_c, S_d, S_e, S_f, S_g,…—the members of each set all satisfying the condition associated with that set. A given set of compossibles, or possibilities mutually satisfying multiple conditions, such as S_γ, is simply the intersection of the sets of possibilities satisfying each individual condition, or equivalently, the set that satisfies the conjunction of those multiple conditions.

The Enumeration of Compossible Sets

As the number of conditions pertaining to manifestation is indeterminate, the number of sets of compossibles will likewise be indeterminate as well. Nevertheless, the notion of how many compossible sets there might be is well understood mathematically. In particular, if the number of conditions is N_C, then the number of sets of possibilities satisfying a single condition will also be N_C. The number of compossible sets may be broken down into those satisfying two conditions, three conditions, four conditions and so on, all the way up to N_C conditions. This may be expressed in terms of the binomial coefficient or "choose function" of combinatoric mathematics for the number of compossible sets satisfying k conditions as:

$$\binom{N_C}{k} \equiv \frac{N_C!}{k!(N_C - k!)}$$

where, for any integer $n > 0$,

$$n! \equiv (1)(2)(3)\ldots(n-2)(n-1)(n)$$

We may sum over values of k from 2 to N_C, corresponding to the multiplicity of conditions satisfied by each grouping of compossible sets, and so arrive at a total number of compossible sets:

$$\sum_{k=2}^{N_C} \binom{N_C}{k}$$

Generally, then, there will be vastly more compossible sets than sets of possibilities satisfying single conditions. However, as these compossible sets are defined in terms of multiple set intersections, they will, in general, be much smaller than sets of possibilities satisfying single conditions. Many compossible sets will be empty, specifically those that satisfy mutually incompatible conditions, equivalent to the intersection of disjoint sets of possibilities.

Compossible Sets and Individual Possibilities

Another way to approach the question of compossibles is by way of consideration of a given individual possibility. Such a possibility will be manifest subject to the set of conditions that are in accord with its own inherent nature. As an extension of the previous analysis concerning roundness, consider, for simplicity, a semi-idealized material object such as a ping-pong ball. The conditions concordant with its nature might include: materiality, extension, temporality, sphericity, whiteness, opaqueness, hollowness and so forth. Such a possibility of manifestation will only be realized when all of its associated conditions are present. Nevertheless, this possibility is an element of each set associated with each individual condition. That is:

$$p_{\text{ping-pong ball}} \in S_{\text{materiality}}, S_{\text{extension}}, S_{\text{temporality}}, \ldots$$

Further, it is also an element of each compossible set formed by the various intersections of the sets associated with each individual condition.

An analogy from analytic geometry may prove helpful: Consider a three dimensional space defined in the conventional way with orthogonal axes x, y and z. Each point in this space—associated with a unique set of values of x, y and z—may be considered as a unique possibility. Any specification upon the value that the variables x, y or z may assume may be considered as a condition. The collection of points satisfying such a condition may be considered as the set of possibilities under that condition. The collection of points satisfying multiple such conditions may be considered as the compossible set under those conditions. Consider a particular point, defined conventionally in terms of orthogonal values x, y and z, such as (–1, 1, 0). The "nature" of this point is concordant with the conditions '$x = -1$', '$y = 1$' and '$z = 0$', and it will be manifest precisely and only where all of its associated conditions are satisfied. One may also identify the set of possibilities corresponding to a given condition, such as '$z = 0$', which in this instance, will simply be the *x-y* plane. The point in question, $(-1, 1, 0)$, lies in this plane, and so is a member of this set. Further, one may identify the compossible set corresponding to multiple conditions, such as '$x = -1$' $\wedge$ '$y = 1$', which will be the line parallel to the *z* axis and passing through the *x-y* plane at $x = -1$, $y = 1$. The point in question, $(-1, 1, 0)$, also lies on this line, and so is a member of this set as well. One may, by extension, conceive other conditions, such as '$y > 0$', as well as other sets of possibles and compossibles based upon these conditions, such that the point in question may be an element of these sets as well. Nevertheless, no matter how many sets of possibles and compossibles this point belongs to, it will be "found" in only one location within the space of our analogy.

A given set of possibles or compossibles may be conceived as a kind of "cut" through the domain of manifestation that will necessarily include some possibilities and exclude others. A given possibility may be included in numerous such "cuts" without ceasing to occupy its own ontological "location" within manifestation; it is here and only here that it is realized. Of necessity, it is also here that the full collection of conditions concordant with its nature will be satisfied.

Compossible Sets as Existent Worlds

A set of compossibles, determined by multiple compatible conditions, is designated, in Guénon's terminology, as a "world", "domain" or "mode" of universal Existence. As Guénon states:

> [If] we consider what we might call a world…, that is, the entire domain formed by a certain ensemble of compossibles realized in manifestation, then these compossibles must be the totality of possibles that satisfy certain conditions characterizing and precisely defining that world… The other possibles, which are not determined by the same conditions and consequently cannot be part of the same world, are obviously no less realizable for all that, but of course each according to the mode befitting its nature.[23]

Again, although Guénon nowhere invokes the specific mathematical language of set theory, his understanding is nevertheless very close to this more mathematically formal expression. Given that a world is precisely formally equivalent to a compossible set, we may abstractly define a given world in these same terms. For a given collection of defining conditions, with subscript indices comprising, say, the set α—such as $\alpha = \{a, f, l, q, \ldots\}$—the world thus defined is:

$$S_\alpha = \bigcap_{i \in \alpha} S_i = \{p \in M : \wedge_{i \in \alpha} C_i\}$$

Note the identity of this general expression to that of a compossible set, as given previously. Of course, just as there are many compossible sets, so there are many worlds within manifestation:

> …if by this term [world] one understands only a certain whole of compossibles, …it is as absurd to say that its existence prevents the coexistence of other worlds as it would be to maintain that the existence of a circle is incompatible with the coexistence of a square, a triangle, or any other figure.[24]

Whether a given possibility belongs to a given world or not depends—as with membership in compossible sets—on the concordance of the nature of that possibility with the conditions delimiting that world:

> …the conditions by which a determinate world is defined…exclude from that world those possibles the nature of which does not imply a realization subject to those same conditions; these possibles are thus outside the limits of the world under consideration, but that in no way excludes them from…the entire domain of universal manifestation.[25]

Just as this is true with respect to the a given world, determined by an ensemble of delimiting conditions, so it is true if only a single delimiting condition is considered in isolation, equivalent to the distinction between compossible and possible sets:

> …instead of considering the totality of the conditions that determine a world, …one could also take the same point of view but consider one of these conditions in isolation; for instance, from among the conditions of the corporeal world we

> might take space, envisaged as what contains spatial possibilities. It is quite evident that by definition only spatial possibilities can be realized in space; but it is no less evident that this does not prevent non-spatial possibilities from being equally realized [i.e. manifested] outside of that particular condition of existence which is space.[26]

In this instance, although Guénon is not explicit in his major metaphysical works,[27] one might take as delimiting conditions determining the "corporeal world" such basic conditions as form, spatiality, temporality, materiality, and perhaps others as well. Particularizing the abstract set-theoretic notation for a world given above, we may formally describe the "corporeal world", $S_{corporeal}$, and its set of conditional subscripts as:

$$corporeal = \{form, spatiality, temporality, materiality\}$$
$$S_{corporeal} = \bigcap_{i \in corporeal} S_i = \{p \in M : \wedge_{i \in corporeal} C_i\}$$

or, equivalently although less compactly, as:

$$\begin{aligned} S_{corporeal} &= S_{form} \cap S_{spatiality} \cap S_{temporality} \cap S_{materiality} \\ &= \{p \in M : C_{form} \wedge C_{spatiality} \wedge C_{temporality} \wedge C_{materiality}\} \end{aligned}$$

As Guénon describes in the passage above, one may consider the condition of spatiality, and the set of possibilities realizable according to this condition, in isolation from the "corporeal world" that it in part determines. This is, of course, simply equivalent to considering a single condition and its associated set of possibilities, in this case:

$$S_{spatiality} = \{p \in M : C_{spatiality}\}$$

The possibilities comprising the set $S_{spatiaity}$ are precisely those whose individual natures are in concordance with the condition of spatiality, apart from the broader question of membership in the "corporeal world" as such. That a given possibility belong to the possible set $S_{spatiaity}$ is a necessary, but not a sufficient condition for it to belong to the "corporeal world", $S_{corporeal}$. For this further, more restrictive membership to be satisfied, the possibility must be realizable under *all* the conditions of the corporeal world. However, if any one of the necessary conditions is not satisfied—such as spatiality—then the possibility is not realizable in that world. Thus, in this instance, a non-spatial possibility is not compatible with the condition of spatiality and thus is also not compatible with the corporeal world, subject to spatiality as one of its governing conditions. Another way of approaching this insight is to note that, as specified earlier, the intersection of multiple sets is necessarily equal to or a subset of each of the sets in question, or, in this instance:

$$S_{corporeal} \subseteq S_{form}, S_{spatiality}, S_{temporality}, S_{materiality}$$

Given this, $S_{corporeal}$ must necessarily be equal to or a subset of $S_{spatiaity}$ and thus since a non-spatial possibility cannot belong to the set $S_{spatiaity}$, it necessarily cannot belong to any subset of $S_{spatiaity}$, including $S_{corporeal}$. Yet another way of approach is to recall that the intersection of the sets of possibles defining the "corporeal world" may be defined directly in terms of set membership as:

$$\begin{aligned} S_{corporeal} &= S_{form} \cap S_{spatiality} \cap S_{temporality} \cap S_{materiality} \\ &= \{p \in M : (p \in S_{form}) \wedge (p \in S_{spatiality}) \wedge \\ &\qquad (p \in S_{temporality}) \wedge (p \in S_{materiality})\} \end{aligned}$$

Since the conjunctive connectives $\wedge$ in the above expression imply that all the membership conditions must be satisfied, the failure to satisfy any one of these conditions—including that of spatiality—implies non-membership in the compossible set $S_{corporeal}$.

Existent Worlds and Their Possible Sets

Guénon's most explicit statement of the relationship between the individual sets of possibles and the compossible set, or world, formed by their conjoint conditions clarifies their relative extension:

> ….the union or combination [of special conditions of existence] determines a world. …it goes without saying that the several conditions thus united must be mutually compatible, and that

> their compatibility obviously entails that of the possibles they include respectively, with the restriction that the possibles subject to the given group of conditions can only constitute a part of those which are comprised in each of the conditions envisaged apart from the others, from which it follows that these conditions in their integrality, beyond what they hold in common, will include various prolongations that nevertheless still belong to the same degree of universal Existence.[28]

Here, he is clearly indicating that the compossible set comprising a world forms a subset of each of the individual sets of possibles, each determined by a single given condition of existence, which will form "various prolongations" beyond that world. Further, just as a given compossible set is formed by the intersection of multiple sets of possibles, so a given possible set and its associated condition may apply to more than a single compossible set:

> …each of these conditions considered in isolation from the others can extend beyond the domain of that modality [i.e. world], and, whether through its own extension or through its combination with different conditions, can then constitute the domain of other modalities…[29]

For instance, if one considers a hypothetical world, $S_{hypothetical}$, where the condition of spatiality applies but not the condition of materiality, such a world will in general be distinct from the corporeal world, $S_{corporeal}$, since its set of defining conditions is not identical. Both worlds, though distinct, will be determined in part by the same spatial condition, $C_{spatiality}$, which may apply to multiple worlds, or sets of compossibles. The associated set of possibles conditioned by spatiality, $S_{spatiaity}$, will then necessarily be a superset of both the corporeal and hypothetical worlds:

$$S_{spatiality} \supseteq S_{corporeal} , S_{hypothetical}$$

Although $S_{corporeal}$ and $S_{hypothetical}$ are both subsets of $S_{spatiaity}$, it is not in general possible to infer from this any specific relation between these two worlds, which may be related in any of the four possible ways specified by set theory: subset/superset, identical, conjoint or disjoint. The specific relationship between the two worlds will be dependent on the nature and interrelationships of their defining conditions.

We may generalize the two immediate relations given above:

For a given world defined as S_α, where α is a set of conditional subscripts $\{a, b, c, d, \ldots\}$ related to conditions C_a, C_b, C_c, C_d,… and associated sets of possibles S_a, S_b, S_c, S_d,… it must hold that:

$$S_\alpha \subseteq S_a, S_b, S_c, S_d, \ldots$$

Similarly, for a given condition C_a and associated set of possibles S_a, for all worlds S_α, S_β, S_γ, S_δ,… determined by C_a such that $a \in \alpha, \beta, \gamma, \delta, \ldots$ it must hold that:

$$S_a \supseteq S_\alpha, S_\beta, S_\gamma, S_\delta, \ldots$$

We may further generalize addition relations as follows:

For a given world defined as S_α, where α is a set of conditional subscripts $\{a, b, c, d, \ldots\}$ related to conditions C_a, C_b, C_c, C_d,… and associated sets of possibles S_a, S_b, S_c, S_d,… if any two of these sets of possibles are disjoint—equivalent to any two conditions being incompatible—the world will be empty:

$$S_\alpha = \phi$$

For a given world S_α, defined by a conditional set α, any world S_β, defined by a conditional set β, such that $\alpha \supseteq \beta$, it must hold that:

$$S_\alpha \subseteq S_\beta$$

Conversely, for a given world S_α, defined by a conditional set α, any world S_β, defined by a conditional set β, such that $\alpha \subseteq \beta$, it must hold that:

$$S_\alpha \supseteq S_\beta$$

Although we do not prove these relations here, all of them may be straightforwardly derived from the fundamental subset relation between the intersection of multiple sets and the sets themselves.

As an example of the two additional general relations immediately above, we may consider the distinction made between Guénon and Frithjof Schuon with respect to the conditions pertaining to the corporeal world. Guénon tentatively asserts that there are five: form, space, time, matter, life; Schuon differs, asserting that the first four mentioned are conditions of corporeality, but not the fifth.[30] Unless life is given a very particular notion as a condition, Schuon would seem to be correct, as, for instance, granite is clearly corporeal, but is not in any typical sense considered living. We might consider a world, S_{animate}, defined by the limiting conditions:

$$animate = \{form, spatiality, temporality, materiality, life\}$$

just as the corporeal world is defined by the limiting conditions:

$$corporeal = \{form, spatiality, temporality, materiality\}$$

Then, since $corporeal \subseteq animate$, we may assert from the above relations that:

$$S_{corporeal} \supseteq S_{animate}$$

Thus, the corporeal world encompasses the animate world, whose possibilities are necessarily circumscribed by it.

General and Special Conditions of Manifestation

In the discussion above, we treat S_{animate} as a world proper, similar to $S_{corporeal}$. This implies an assumption regarding the status of the condition of life, namely that it is what might be termed a *general condition*. As a contrasting case, consider the condition of sphericity: one could, in principle, define a spherical world, delimited by all the conditions of corporeality plus that of sphericity. However, this would be a very odd notion of a world; a much more sensible and natural approach would be to consider sphericity, not as a general condition, but rather as a *special condition*. In this approach, we might well define a compossible set delimited by all the conditions of corporeality plus that of sphericity, but such a set is simply a collection of spherical possibilities contained within the corporeal world. This distinction imposes at once a twofold segregation between conditions as well as a twofold segregation between compossible sets. Thus, a given condition may be either a general or special condition, whereas a given compossible set may be either a world or a compossible set within a world proper.

The distinction between general and special conditions may be inferred from Guénon in passing, as when he refers to:

> …the distinction one could establish when one is no longer referring to universal manifestation in its integrality, but simply to one or another of the general or special conditions of existence known to us.[31]

Normally, however, Guénon does not employ this particular distinction, often referring to *all* conditions of manifestation as "special conditions". Schuon, in addressing Guénon's schema, clearly does distinguish between general conditions and "secondary categories"—equivalent to what we here term special conditions—among which he explicitly includes both life and color as examples. We employ the term *subworld* to refer to any compossible set that is not a world proper. We may formally distinguish between a world and subworld as follows:

A world is a compossible set in which all its defining conditions are general conditions; a subworld is a compossible set in which at least one of its defining conditions is a special condition.

Just as the distinction between general and special conditions imposes a segregation on compossible sets, so it also imposes a segregation on individual sets of possibilities determined by single conditions. In particular, we may formally distinguish between what we term a *general possible set* and a *special possible set* as follows:

A general possible set is a set of possibilities delimited by a general condition; a special possible set is a set of possibilities delimited by a special condition.

It then follows that a world is necessarily the intersection only of general possible sets, whereas a subworld is the intersection of sets of possibilities, at least one of which is a special possible set.

If every compossible set solely delimited by general conditions is a world, then for every subworld, there must be at least one world in relation to which it is a subset. This is so, since the set of conditions for a subworld will necessarily include both general and special conditions, in relation to which it is always possible to specify a set of conditions including all or some of the general conditions in question and thus forming a subset of the overall set of conditions in question. It then follows from the general relation previously specified that the subworld must form a subset of any of the worlds defined by any partial or complete set of the general conditions in question. As a case of interest, we consider the world delimited by the set of all general conditions partially delimiting the subworld in question. We might term this the *immediate world* of the subworld in question. Thus, for example, the corporeal world is the immediate world of the subworld comprised of spherical corporeal possibilities—those satisfying the general conditions of corporeality as well as the special condition of sphericity—which in turn is a subset of this world.

The distinction between general and special conditions will often be, but is not always evident. It is fairly clear that conditions such as space and time are properly defined as general conditions, while conditions such as shape and color are properly defined as special conditions. A condition such as life, however, is not so evident in its classification; the determination could reasonably be argued either way without necessarily arriving at a satisfactory conclusion. The consequence of this potential ambiguity is that the bifurcating distinction between general and special conditions—along with the associated bifurcation between worlds and subworlds as well as general and special possible sets—while valid in principle, may be somewhat arbitrary in application. Ultimately, the matter hinges on the manner in which one distinguishes general conditions and special conditions. Neither Guénon nor Schuon address this issue, but we might tentatively suggest as a discriminating determination that if a condition of manifestation may be considered as a modification of a more general condition, then it should be defined as a special condition. For instance, shape may be considered a modification of space, while color may be considered a modification of matter. In contrast, space, time and matter are *sui generis*, and cannot be considered as modifications of any more general condition. Under this definition, life might well be considered a modification of matter, thus determining life as a special condition; alternatively, life might also be considered *sui generis*, as something that participates in matter but is not of the nature of matter, and thus a general condition in its own right.

Existent Worlds and Degrees of Existence

Guénon is fairly clear and consistent in his description of a world or domain of manifestation, but is unfortunately less so in his description of a key correlative concept: a degree of universal Existence. In certain places, he seems to identify degrees of existence with worlds themselves, as when he writes of: "…groups of possibilities corresponding to one of the 'worlds' or degrees…"[32] Similarly, and even more explicitly, he states:

> When we speak of Existence, we thus mean universal manifestation, with all the states or degrees that it contains, each of which may equally be described as a 'world', one of a series that is indefinite in its multiplicity.[33]

Somewhat more frequently, however, he asserts a different relationship between worlds and degrees of existence. For instance, in respect to the corporeal world, Guénon clearly envisages this as a subset of a specific degree of existence:

> …the corporeal world, which is entirely situated at one single degree of Existence and represents only a quite restricted portion of that.[34]

Further, in respect to space, one of the conditions of the corporeal world, he envisages this condition—and by extension its associated possible set—as also being bounded by this same degree of existence:

> …space, even when envisaged in the whole extension it is capable of, is no more than a special condition which is contained in one of the degrees of universal Existence, and to which (added to or combined with other conditions of the same order) certain of the multiple domains [i.e. worlds] comprised in that degree of Existence are subjected…[35]

Finally, he envisages a given degree of Existence as not only containing a particular world, but in fact comprising a sum total of such worlds:

> The sum total of the domains [i.e. worlds]—indefinite in extent—that contain all the modalities of one and the same individuality constitutes one degree of universal Existence…[36]

To review, a degree of existence contains certain individual conditions as well as the worlds that they in part determine; further, such a degree may be considered as a totality of all the worlds that it contains. Reexpressing this in set-theoretic terms, we may say that sets of possibles, associated with given conditions, are subsets of a given degree of existence; further, the worlds formed by the intersections of such possible sets are also subsets of the same degree of existence; additionally, the degree of existence is the union of all such worlds. To formalize this, a given degree of existence, S_{D1}, contains conditions (where, to simplify the discussion, we shall only consider general conditions):

$$C_a, C_b, C_c, \ldots C_x, C_y, C_z$$

where the conditional subscripts are arbitrary and represent an arbitrary number of conditions. This degree will then also contain associated sets of possibilities:

$$S_a, S_b, S_c, \ldots S_x, S_y, S_z$$

Various worlds may be determined through the delimitation of multiple conditions, as described previously. Since each above condition and associated possible set is singly contained within the degree of existence, it follows that all these conditions and associated possible sets are so contained. That is:

$$S_{D1} \supseteq S_a, S_b, S_c, \ldots S_x, S_y, S_z$$

If we consider other such degrees, each with its own collection of conditions and possible sets, these sets must also be so contained within their own degrees. This in turn implies that each degree include its own possible sets, but not those of any other degree, since each degree fully contains each of its possible sets. The only manner in which this may be satisfied is for the degree to comprise the union of its possible sets, as this relation will encompass those sets without including any sets apart from these. That is:

$$S_{D1} = S_a \bigcup S_b \bigcup S_c \bigcup \ldots S_x \bigcup S_y \bigcup S_z$$

Alternatively, if we define a set of all conditional subscripts pertaining to the degree in question:

$$\omega = \{a, b, c, \ldots x, y, z\}$$

we may define this more compactly as:

$$S_{D1} = \bigcup_{i \in \omega} S_i = \{p \in M : \vee_{i \in \omega} C_i\}$$

where i is an index variable evaluated over the members of the set of conditional subscripts and the union and disjunction operators are treated as summations. Note that this is in some sense precisely the inversion of the world delimited by all the conditions pertaining to this degree:

$$S_\omega = \bigcap_{i \in \omega} S_i = \{p \in M : \wedge_{i \in \omega} C_i\}$$

Here, the degree of existence, S_{D1}, encompasses the possibilities across all its possible sets, whereas the maximally delimited world within this degree, S_ω, only encompasses those possibilities common to all sets of possibles within the degree, and in many instances will be empty. The first is maximally inclusive relevant to the conditions in question, the second maximally restrictive.

The limitation inherent in the extension of sets of possibles associated with a given degree is quite different in character than the extension of such sets as associated with a given world. A world is by definition a set intersection of multiple sets of possibles so that in general a set of possibles will extend in part beyond the world in question, which only pertains to its region of intersection. A degree, in contrast, is entirely inclusive of its sets of possibles, which have no extension beyond the degree itself. This in turn implies that there is no overlap of sets of possibles across multiple degrees of existence: in terms of possible sets, each degree is an island. This relation between possible sets and degrees is an assertion of Guénon's metaphysics. It specifically implies that the collection of possible sets comprising a given degree, when taken as a union or whole, are disjoint to any other sets of possibles within manifestation. The same may be asserted with respect to the worlds formed of these possible sets. Ultimately, both the sets of possibles and the worlds formed of them are isolated to a given degree as a consequence of the boundedness of the associated delimiting conditions to that degree. We may formalize this isolation or disjointness of degrees by asserting that for any two degrees of existence, S_{Dm} and S_{Dn}:

$$S_{Dm} \cap S_{Dn} = \phi$$

Guénon implies that a degree of existence is a "sum total" of a given collection of worlds, which we above tentatively interpreted as inferring that such a degree could be considered as a set-theoretic union of all worlds that could be formed from the conditions within that degree. However, this is not quite correct, as the union of all sets of possibles associated with these conditions is not coextensive with the union of all worlds that may be formed by the various intersections of these sets. To demonstrate this, consider the region of each set of possibles that does not intersect any other set of possibles: any possibility in such a region with be determined by only a single condition, but since a world is by definition determined by multiple conditions, such a possibility is not contained in any world, but *is* contained in the degree in question since its set of possibles is so contained. This observation carries larger implications insofar as it clarifies that there are in principle possibilities of manifestation that are not a part of any world or subworld. The union of all worlds formed by the conditions within a degree of existence must thus necessarily be a subset of the union of all sets of possibles associated with these same conditions, which is in turn coextensive with the degree of existence itself.

The formal expression for the union of all worlds is in principle rather complicated, as it involves the listing of all worlds formed from two conditions, three conditions and so on. However, we have already established as a general relation that any world whose conditions are a superset of those of another world will necessarily be a subset of that world. Since any world determined by three or more conditions will necessarily have a conditional set that is a superset of the conditional set of one of the worlds determined by only two conditions, it follows that any such world will be a subset of one or more of the worlds determined by only two conditions. Since the union of a set and its subset simply yields the initial set, it follows that in determining the union of all possible worlds in a given degree of existence, we need in practice only consider the collection of worlds delimited by only two conditions. We may express this union of worlds formally as:

$$\bigcup_{\substack{i\in\omega \\ j>i}} \left(S_i \cap S_j\right)$$

where *i* and *j* are both index variables evaluated over the members of the set of conditional subscripts and the union operator is treated as a summation. The term in parentheses is a generalized bi-conditional world formed from the intersection of two sets of possibles whose conditions are given by *i* and *j*. These index variables are run over the ordered set of conditional subscripts in ω, where *i* successively takes on all conditional subscript values and *j* takes on all those values following *i* in the set of conditional subscripts. This combined enumeration represents all the unique pairs of conditions in the degree of existence. The number of terms in the multiple union of worlds will simply be the number of such pairs: if there are N_ω conditions pertaining to the degree of existence in question, then the number of terms in the union will simply be $N_\omega(N_\omega - 1)/2$, the general mathematical expression for the number of unique pairs.

With the formal expressions for a degree of existence, the union of the sets of possibles associated with conditions contained within this degree and the union of the sets of worlds formed from these same conditions, we may formally express the relations between each as:

$$S_{D1} = \bigcup_{i\in\omega} S_i \supseteq \bigcup_{\substack{i\in\omega \\ j>i}} \left(S_i \cap S_j\right)$$

As elucidated by Guénon, degrees of existence are the most significant structures or assemblages found within the manifest, along a scale that runs from individual possibilities, to subworlds, to worlds, to sets of possibles, to degrees and at last to the domain of manifestation in its entirety. With this set-theoretic treatment of degrees of existence, we have at last addressed the entire scale of manifestation envisaged by Guénon in the context of its set-theoretic description. As such, the demonstration of the equivalence of this description for the expression and clarification of Guénon's metaphysics now stands complete.

Conclusion

In the context of manifestation—which has largely bound our treatment of Guénon's metaphysics—in what regard may his metaphysical exposition be considered veracious? In terms of the axioms of his structure of thought, he assumes very little: discreteness and particularity, collection and membership, attribution and conditionality. These axioms may be considered at once fundamental and sufficient, being profoundly primary to our most innate experience of the world: things appear as discrete and multiple, they may or may not stand in mutual relatedness or cohere into larger wholes, they may or may not express certain attributes or be delimited by certain conditions. Any such thing may be considered a possibility that has been manifested, all manifested possibilities in their entirety—the "ten thousand things"—by definition comprising the domain of manifestation. There is little purchase upon which one might critique these very spare axiomatic assumptions; yet, spare though they may be, out of them Guénon fashions worlds.

Apart from his axiomatic foundations, one must also consider the coherence of his intellectual structure; insofar as this structure conforms to the set-theoretic description we have elucidated, it possesses the same sufficient coherence as the mathematical description itself. Further, as we have demonstrated, it is possible to leverage the coherence and "truth" of this mathematical description to extend Guénon's metaphysical vision, even to the point of asserting general relationships with confidence, although they are nowhere developed by Guénon himself. In this, we find that the language of set theory is adequate as a valid mode of expression for the direct description of metaphysical realities within the domain of manifestation.

This description proceeds in a different manner than that of symbolism, which does not speak directly of metaphysical realities, but rather indirectly through phenomena to the metaphysical realities that stand "behind" them. It is even more direct than other mathematical descriptions that Guénon employs, such as the geometric description of the vertical axis in *The Symbolism of the Cross*, which is at once symbolic in the traditional character of this term as well as a direct mathematical description. Although the clarification of his metaphysics through a set-theoretic description does lend a certain degree of particularity to his exposition, the generality and abstraction of his exposition to a large extent remains. This is in large part inevitable, given the character of metaphysics itself, just as Plato nowhere attempts an exhaustive cataloging of the Forms and their relationships.

As a final observation, it is interesting and perhaps deeply instructive to note the close similarity of Guénon's metaphysical description of manifestation to certain traditional ontologies developed through close examination of discreteness and conditionality in the context of existence, particularly the Buddhist Abhidhamma and Madhyamika schools[37], which, in a sense, make these fundamental characteristics of existence the touchstones of their philosophical systems. This similarity is somewhat ironic, as Guénon—following orthodox Hinduism—considered Buddhism a heterodox deviation for much of his life, until his conversion on this point by Ananda Coomaraswamy and Marco Pallis. We emphasize that this similarity holds only at the level of manifestation; insofar as his overall metaphysics is concerned, he is, of course, most closely resonant with the Vedanta.

Regardless of such similarities, Guenon's metaphysical *summa* stands on its own, at once one of the most comprehensive and penetrating restatements of traditional metaphysics of the present era, while also marked by his own particular genius, one shaped, in no small measure, in the light of mathematical truth.

Notes:

1. René Guénon, *The Metaphysical Principles of the Infinitesimal Calculus* (Hillsdale, NY: Sophia Perennis, 2003), p. 130.
2. Frithjof Schuon, *René Guénon: Some Observations* (Hillsdale, NY: Sophia Perennis, 2004), p. 7.
3. See, for instance, Paul Chacornac, *The Simple Life of René Guénon* (Hillsdale, NY: Sophia Perennis, 2004).
4. See, for instance, Teun Koetsier & Luc Bergmans, eds., *Mathematics and the Divine: A Historical Study* (London: Elsevier, 2005). Ch. 33 of this book is the essay "Symbol and Space According to René Guénon," by Bruno Pinchard.
5. So termed to distinguish it from axiomatic set theory, which was subsequently developed.
6. See, for instance, http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Beginnings_of_set_theory.html (accessed Sept. 1st, 2006)
7. Chacornac, op. cit., p. 16.
8. Guénon, *The Metaphysical Principles of the Infinitesimal Calculus*, p. 16.
9. See, for instance, http://plato.stanford.edu/entries/set-theory/ (accessed Sept. 1st, 2006)
10. Guénon, *The Metaphysical Principles of the Infinitesimal Calculus*, p. 118.
11. René Guénon, *Man and His Becoming according to the Vedanta* (Hillsdale, NY: Sophia Perennis, 2001), p. 151.
12. Robin Waterfield, *René Guénon and the Future of the West: The Life and Writings of a 20th-Century Metaphysician* (Hillsdale, NY: Sophia Perennis, 2002), pp. 62-3.
13. René Guénon, *The Multiple States of the Being* (Hillsdale, NY: Sophia Perennis, 2001), p. 7.
14. Ibid., p. 9.
15. Ibid., p. 11.
16. Ibid., p. 9.
17. Ibid., p. 20.

18. Ibid., p. 15.
19. Ibid., p. 31.
20. Ibid., p. 28.
21. Ibid., p. 14.
22. Ibid., pp. 14-5.
23. Ibid., p. 15.
24. Ibid., p. 16.
25. Ibid., pp. 16-7.
26. Ibid., pp. 17-8.
27. Guénon does discuss this matter in a separate essay, "The Conditions of Corporeal Existence"; see fn. 30 below.
28. Guénon, *The Multiple States of the Being*, pp. 18-9.
29. Ibid., p. 29.
30. For Guénon's treatment of these conditions, see René Guénon, "The Conditions of Corporeal Existence," *Miscellanea* (Hillsdale, NY: Sophia Perennis, 2003), p. 90; for Schuon's critique of Guénon's five conditions, see Schuon, op. cit., pp. 17-8.
31. Guénon, *The Multiple States of the Being*, pp. 18-9.
32. René Guénon, *The Symbolism of the Cross* (Hillsdale, NY: Sophia Perennis, 2001), p. 17.
33. Ibid., p. 9.
34. Ibid., p. 40.
35. Ibid., p. 97.
36. Ibid., p. 67.
37. Gail Birnbaum, private communication.